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No. of PhD project: IEDS/2023/IM/03

Title of PhD project: ***Mechanical control systems - feedback equivalence, invariants, and applications***

The leading unit: Institute of Mathematics, Polish Academy of Sciences (IM PAN), Warsaw

Requirements:

1. Master degree in mathematics or related fields (e.g. computer science, physics); research topics related to the theory of differential equations or differential geometry.
2. A candidate should be familiar with: methods for solving partial differential equations of the first order; differential manifolds; differential forms on manifolds and their basic properties.
3. English language on the level enabling communication, reading and writing scientific papers. Basic programming skills.

Tasks description:

1. Study of the feedback equivalence problem for the control mechanical systems; characterization of mechanical systems in the class of control affine systems.
2. Study of geometric structures related to mechanical systems.
3. Applications to stability and optimality of solutions of control systems.
4. Study of the literature and the latest available publications on the subject of the project.
5. Writing scientific articles and preparing conference presentations.
6. Regular progress reports.

Abstract:

A control system can be defined as a system of ordinary differential equations in the form

$$x' = F(t, x, u)$$

depending on parameters referred to as control functions. $u = u(t) \in U$ Set U of admissible controls is usually given as a subset of the Euclidean space, e.g.: a subspace, a bounded subset, or a discrete set. A trajectory of a control system system is, by definition, a solution (t) of the system of differential equations obtained for a fixed control function $u(t)$ in the considered class. Classical problems related to the control systems include: controllability (is there a trajectory joining to given states x and y?), optimality (is a trajectory of the system optimal with respect to a given functional?), stability of solution (how the optimal solutions behave under small perturbations?). The control systems naturally appear in applications, for instance: control systems for vehicles and in robotics, models in economy, ecology, biology, and others.

A class of mechanical systems is a particular subclass of general control systems. It is an important and challenging research area that includes, on the one hand, methods of classical mechanics (Lagrangian and Hamiltonian approaches, Lie groups methods, e.t.c.), and modern methods of nonlinear control on the other hand. The state space of a mechanical system is of dimension $2n$ (coordinates correspond to positions x and velocities y) and a system can be written as

$$y_i' = \sum_{jk} A_{ijk}(x) y_j y_k + \sum_j B_{ij}(x) y_j + C_i(x) + \sum_j u_j G_j(x), \quad x_i' = y_i,$$

for certain coefficient functions A, B, C depending on x. The main objective of the research project is to characterize the mechanical systems in the class of the control-affine systems of the form

$$F(t, x, u) = f(x) + \sum_{i=1}^{m} u_i g_i(x).$$

for certain vector fields: f (called drift) and g (called control vector fields). We ask whether a i given control-affine system can be transformed into a mechanical one by means of the feedback transformations:

$$x := X(x) \qquad u := U(x, u)$$

(i.e. the transformation of the control functions depends on the position). It is a hard and open problem in the area of control systems.

Our aim is to apply a geometric approach related to invariants of control systems generalizing the classical notion of curvature. Depending on a progress and interests of a student further research tasks may include:

1. specification of the original problem to a subclass of linear mechanical systems;
2. study of the geometric side of the problem - connections with the conformal and projective geometry;
3. applications to the problems of optimality and stability of solutions (either for general classes of system, or for particular real-life systems), in particular estimates for conjugate points.

Literature:

1. A. M. Bloch, "Nonholonomic Mechanics and Control," Springer-Verlag, New York, 2003.
2. F. Bullo, A. D. Lewis, "Geometric Control of Mechanical Systems," Springer Verlag, New York, 2004.
3. W. Respondek, S. Ricardo, "When is a control system mechanical?", Journal of Geometric Mechanics, Volume 2, Issue 3: 265-302 (2010).

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Other information:

The work will be carried out under supervision of: dr hab. Wojciech Kryński, prof. IM PAN, krynski@impan.pl, Institute of Mathematics of the Polish Academy of Sciences.

Secretary of the IEDS Recruitment Committee: +48 32 3689 380, e-mail: polarknow@us.edu.pl

Information on the IEDS admissions: https://www.mssd.us.edu.pl/en/admission_2023_2024-reg/